MECHANICAL DESIGN · DERIVATION

Deriving the Cycloidal Rotor Profile

From the ring-pin–centre pitch curve to the roller-offset parametric equations.

Benjamin Limberg · Mechanical design

Given · N ring pins · R pin-circle radius · E eccentricity · R_r roller radius · lobes = N − 1

The rotor edge is a curve held exactly one roller radius $\mathbf{R_r}$ inside a base **pitch curve**, measured along that curve's normal. Four moves get us there: write the pitch curve, take its normal, compress that normal into a single angle ψ, then step inward by $\mathbf{R_r}$.

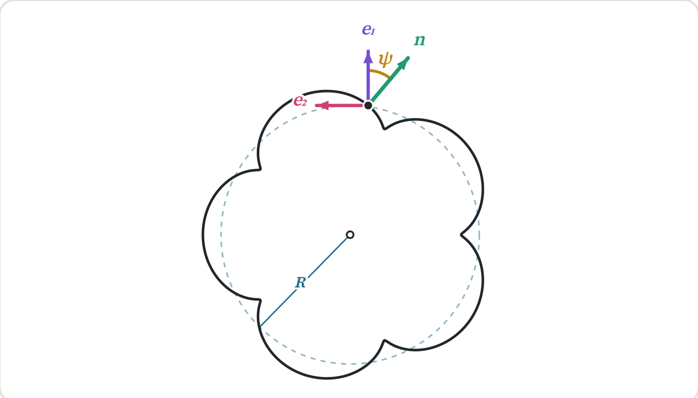

Fig 1 — The normal and the angle ψ. At a point on the pitch curve, the outward normal n is read inside a frame (e_1, e_2) that rotates with t; its heading in that frame is the contact angle ψ.

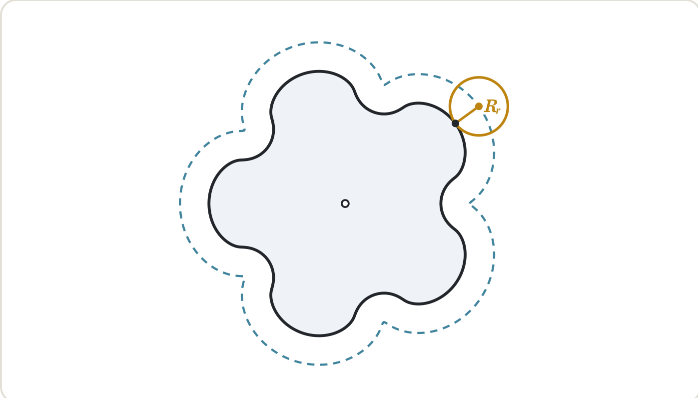

Fig 2 — The roller offset. The rotor profile (solid) is the pitch curve (dashed, the ring-pin–centre path) pushed inward by one roller radius R_r along that normal.

STEP 1 · THE PITCH CURVE

This is the path the ring-pin centres trace in the rotor's own frame — an epitrochoid. The R-terms set a point on the base circle of radius R; the E-terms add the N-fold eccentric wobble that carves the lobes. The leading minus on y fixes the trace orientation.

$$P(t) = \begin{bmatrix} R\cos t - E\cos(Nt) \\ -R\sin t + E\sin(Nt) \end{bmatrix}$$

STEP 2 · THE TANGENT

Differentiating $P(t)$ term by term gives the velocity — the local direction of travel. The chain rule drops a factor of N onto each eccentric term, $\mathrm{d}/\mathrm{d}t[E\sin Nt] = EN\cos Nt$, which is where every EN comes from.

$$T(t) = P'(t) = \begin{bmatrix} -R\sin t + EN\sin(Nt) \\ -R\cos t + EN\cos(Nt) \end{bmatrix}$$

STEP 3 · THE NORMAL

The pin contacts the curve along its normal, so rotate the tangent a quarter turn: $(x', y') \to (-y', x')$. This vector points outward from the curve — subtracting along it later moves us inward, toward the rotor body.

$$N(t) = \begin{bmatrix} R\cos t - EN\cos(Nt) \\ -R\sin t + EN\sin(Nt) \end{bmatrix}$$

STEP 4 · A ROTATING FRAME

To turn that normal into a single number, we read it in a frame that spins with t. e_1 is the reference axis and e_2 is e_1 turned a quarter turn; the normal's angle within this frame is ψ.

$$e_1 = (\cos t,\ -\sin t)$$
$$e_2 = (-\sin t,\ -\cos t)$$

STEP 5 · PROJECT ONTO THE FRAME

The two dot products are the normal's components. In $N \cdot e_1$ the pure-R terms collapse via $\cos^2 + \sin^2 = 1$ and the eccentric terms fuse through $\cos(A-B)$; in $N \cdot e_2$ the R-terms cancel, leaving a lone sine. (Writing $q = EN/R$ makes the algebra fall out.)

$$N \cdot e_1 = R - EN\cos((1-N)t)$$
$$N \cdot e_2 = EN\sin((1-N)t)$$

STEP 6 · THE CONTACT ANGLE ψ

The normal's direction is the arctangent of $N\cdot e_2$ over $N\cdot e_1$. Dividing top and bottom by EN clears the scale, so ψ rides on the single ratio $R/(EN)$ — the heading the offset must follow at every t.

$$\psi(t) = \tan^{-1}\left[\frac{\sin((1-N)t)}{R/(EN) - \cos((1-N)t)}\right]$$

STEP 7 · OFFSET INWARD BY R_R

The rotor surface lies one roller radius R_r inside the pitch curve, along that normal. Expressed through ψ the unit normal is $(\cos(t+\psi), -\sin(t+\psi))$, so we subtract R_r of it from $P(t)$. The y-term gains a **plus** because the normal's y-component is already negative.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = P(t) - R_r(\cos(t+\psi), -\sin(t+\psi))$$

ROTOR PROFILE — THE EQUATIONS THE GENERATOR EXPORTS

$$X(t) = R\cos t - R_r\cos(t+\psi) - E\cos(Nt)$$

$$Y(t) = -R\sin t + R_r\sin(t+\psi) + E\sin(Nt)$$

Sweep t from 0 to 2π to trace one complete disc.